

Keystone Exams: Algebra I

Assessment Anchors and Eligible Content



Pennsylvania Department of Education

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General Introduction to the Keystone Exam Assessment Anchors

Introduction

Since the introduction of the Keystone Exams, the Pennsylvania Department of Education (PDE) has been working to create a set of tools designed to help educators improve instructional practices and better understand the Keystone Exams. The Assessment Anchors, as defined by the Eligible Content, are one of the many tools the Department believes will better align curriculum, instruction, and assessment practices throughout the Commonwealth. Without this alignment, it will not be possible to significantly improve student achievement across the Commonwealth.

How were Keystone Exam Assessment Anchors developed?

Prior to the development of the Assessment Anchors, multiple groups of PA educators convened to create a set of standards for each of the Keystone Exams. Enhanced Standards, derived from a review of existing standards, focused on what students need to know and be able to do in order to be college and career ready. (Note: Since that time, PA Core Standards have replaced the Enhanced Standards and reflect the college- and career-ready focus.) Additionally, the Assessment Anchors and Eligible Content statements were created by other groups of educators charged with the task of clarifying the standards assessed on the Keystone Exams. The Assessment Anchors, as defined by the Eligible Content, have been designed to hold together, or *anchor*, the state assessment system and the curriculum/instructional practices in schools.

Assessment Anchors, as defined by the Eligible Content, were created with the following design parameters:

- **Clear:** The Assessment Anchors are easy to read and are user friendly; they clearly detail which standards are assessed on the Keystone Exams.
- **Focused:** The Assessment Anchors identify a core set of standards that could be reasonably assessed on a large-scale assessment; this will keep educators from having to guess which standards are critical.
- **Rigorous:** The Assessment Anchors support the rigor of the state standards by assessing higher-order and reasoning skills.
- **Manageable:** The Assessment Anchors define the standards in a way that can be easily incorporated into a course to prepare students for success.

How can teachers, administrators, schools, and districts use these Assessment Anchors?

The Assessment Anchors, as defined by the Eligible Content, can help focus teaching and learning because they are clear, manageable, and closely aligned with the Keystone Exams. Teachers and administrators will be better informed about which standards will be assessed. The Assessment Anchors and Eligible Content should be used along with the Standards and the Curriculum Framework of the Standards Aligned System (SAS) to build curriculum, design lessons, and support student achievement.

The Assessment Anchors and Eligible Content are designed to enable educators to determine when they feel students are prepared to be successful in the Keystone Exams. An evaluation of current course offerings, through the lens of what is assessed on those particular Keystone Exams, may provide an opportunity for an alignment to ensure student preparedness.

How are the Assessment Anchors organized?

The Assessment Anchors, as defined by the Eligible Content, are organized into cohesive blueprints, each structured with a common labeling system that can be read like an outline. This framework is organized first by module, then by Assessment Anchor, followed by Anchor Descriptor, and then finally, at the greatest level of detail, by an Eligible Content statement. The common format of this outline is followed across the Keystone Exams.

Here is a description of each level in the labeling system for the Keystone Exams:

- **Module:** The Assessment Anchors are organized into two thematic modules for each of the Keystone Exams. The module title appears at the top of each page. The module level is important because the Keystone Exams are built using a module format, with each of the Keystone Exams divided into two equal-size test modules. Each module is made up of two or more Assessment Anchors.
- **Assessment Anchor:** The Assessment Anchor appears in the shaded bar across the top of each Assessment Anchor table. The Assessment Anchors represent categories of subject matter that anchor the content of the Keystone Exams. Each Assessment Anchor is part of a module and has one or more Anchor Descriptors unified under it.
- **Anchor Descriptor:** Below each Assessment Anchor is a specific Anchor Descriptor. The Anchor Descriptor level provides further details that delineate the scope of content covered by the Assessment Anchor. Each Anchor Descriptor is part of an Assessment Anchor and has one or more Eligible Content statements unified under it.
- **Eligible Content:** The column to the right of the Anchor Descriptor contains the Eligible Content statements. The Eligible Content is the most specific description of the content that is assessed on the Keystone Exams. This level is considered the assessment limit and helps educators identify the range of the content covered on the Keystone Exams.
- **PA Core Standards:** In the column to the right of each Eligible Content statement is a code representing one or more Pennsylvania Core Standard that correlate to the Eligible Content statement. Some Eligible Content statements include annotations that indicate certain clarifications about the scope of an Eligible Content.
 - o “e.g.” (“for example”)—sample approach, but not a limit to the Eligible Content
 - o “i.e.” (“that is”)—specific limit to the Eligible Content
 - o “Note”—content exclusions or definable range of the Eligible Content

How do the K–12 Pennsylvania Core Standards affect this document?

Assessment Anchors and Eligible Content statements are aligned to the PA Core Standards; thus, the former enhanced standards are no longer necessary. Within this document, all standard references reflect the PA Core Standards.

ASSESSMENT ANCHOR			
A1.1.1 Operations with Real Numbers and Expressions			
Anchor Descriptor	Eligible Content		PA Core Standards
A1.1.1.1 Represent and/or use numbers in equivalent forms (e.g., integers, fractions, decimals, percents, square roots, and exponents).	A1.1.1.1.1 Compare and/or order any real numbers. <u>Note:</u> Rational and irrational may be mixed.		CC.2.1.8.E.1 Distinguish between rational and irrational numbers using their properties.
	A1.1.1.1.1.2 Simplify square roots (e.g., $\sqrt{24} = 2\sqrt{6}$).		CC.2.1.8.E.4 Estimate irrational numbers by comparing them to rational numbers. CC.2.1.HS.F.1 Apply and extend the properties of exponents to solve problems with rational exponents. CC.2.1.HS.F.2 Apply properties of rational and irrational numbers to solve real-world or mathematical problems.
Anchor Descriptor	Eligible Content		PA Core Standards
A1.1.1.2 Apply number theory concepts to show relationships between real numbers in problem-solving settings.	A1.1.1.2.1 Find the Greatest Common Factor (GCF) and/or the Least Common Multiple (LCM) for sets of monomials.		CC.2.1.6.E.3 Develop and/or apply number theory concepts to find common factors and multiples. CC.2.1.HS.F.2 Apply properties of rational and irrational numbers to solve real-world or mathematical problems.

Eligible Content may be assessed using problem-solving situations.

Anchor Descriptor	Eligible Content	PA Core Standards
A1.1.1.3 Use exponents, roots, and/or absolute values to solve problems.	A1.1.1.3.1 Simplify/evaluate expressions involving properties/laws of exponents, roots, and/or absolute values to solve problems. <u>Note:</u> Exponents should be integers from -10 to 10 .	CC.2.1.HS.F.1 Apply and extend the properties of exponents to solve problems with rational exponents. CC.2.1.HS.F.2 Apply properties of rational and irrational numbers to solve real-world or mathematical problems. CC.2.2.8.B.1 Apply concepts of radicals and integer exponents to generate equivalent expressions.
Anchor Descriptor	Eligible Content	PA Core Standards
A1.1.1.4 Use estimation strategies in problem-solving situations.	A1.1.1.4.1 Use estimation to solve problems.	CC.2.2.7.B.3 Model and solve real-world and mathematical problems by using and connecting numerical, algebraic, and/or graphical representations. CC.2.2.HS.D.9 Use reasoning to solve equations and justify the solution method.
Anchor Descriptor	Eligible Content	PA Core Standards
A1.1.1.5 Simplify expressions involving polynomials.	A1.1.1.5.1 Add, subtract, and/or multiply polynomial expressions (express answers in simplest form). <u>Note:</u> Nothing larger than a binomial multiplied by a trinomial.	CC.2.2.HS.D.1 Interpret the structure of expressions to represent a quantity in terms of its context. CC.2.2.HS.D.2 Write expressions in equivalent forms to solve problems.
	A1.1.1.5.2 Factor algebraic expressions, including difference of squares and trinomials. <u>Note:</u> Trinomials are limited to the form $ax^2 + bx + c$ where a is equal to 1 after factoring out all monomial factors.	CC.2.2.HS.D.3 Extend the knowledge of arithmetic operations and apply to polynomials. CC.2.2.HS.D.5 Use polynomial identities to solve problems.
	A1.1.1.5.3 Simplify/reduce a rational algebraic expression.	CC.2.2.HS.D.6 Extend the knowledge of rational functions to rewrite in equivalent forms.

Eligible Content may be assessed using problem-solving situations.

ASSESSMENT ANCHOR		
A1.1.2 Linear Equations		
Anchor Descriptor	Eligible Content	PA Core Standards
A1.1.2.1 Write, solve, and/or graph linear equations using various methods.	A1.1.2.1.1 Write, solve, and/or apply a linear equation (including problem situations).	CC.2.1.HS.F.3 Apply quantitative reasoning to choose and interpret units and scales in formulas, graphs, and data displays.
	A1.1.2.1.2 Use and/or identify an algebraic property to justify any step in an equation-solving process. <u>Note:</u> Linear equations only.	CC.2.1.HS.F.4 Use units as a way to understand problems and to guide the solution of multi-step problems.
	A1.1.2.1.3 Interpret solutions to problems in the context of the problem situation. <u>Note:</u> Linear equations only.	CC.2.1.HS.F.5 Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.
		CC.2.2.8.C.1 Define, evaluate, and compare functions.
		CC.2.2.8.C.2 Use concepts of functions to model relationships between quantities.
		CC.2.2.HS.C.3 Write functions or sequences that model relationships between two quantities.
		CC.2.2.HS.D.7 Create and graph equations or inequalities to describe numbers or relationships.
		CC.2.2.HS.D.8 Apply inverse operations to solve equations or formulas for a given variable.
		CC.2.2.HS.D.9 Use reasoning to solve equations and justify the solution method.
		CC.2.2.HS.D.10 Represent, solve, and interpret equations/inequalities and systems of equations/inequalities algebraically and graphically.

Eligible Content may be assessed using problem-solving situations.

Anchor Descriptor	Eligible Content	PA Core Standards
A1.1.2.2 Write, solve, and/or graph systems of linear equations using various methods.	A1.1.2.2.1 Write and/or solve a system of linear equations (including problem situations) using graphing, substitution, and/or elimination. <u>Note:</u> Limit systems to two linear equations.	CC.2.1.HS.F.5 Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.
	A1.1.2.2.2 Interpret solutions to problems in the context of the problem situation. <u>Note:</u> Limit systems to two linear equations.	CC.2.2.8.B.3 Analyze and solve linear equations and pairs of simultaneous linear equations. CC.2.2.HS.D.7 Create and graph equations or inequalities to describe numbers or relationships. CC.2.2.HS.D.9 Use reasoning to solve equations and justify the solution method. CC.2.2.HS.D.10 Represent, solve, and interpret equations/inequalities and systems of equations/inequalities algebraically and graphically.

Eligible Content may be assessed using problem-solving situations.

ASSESSMENT ANCHOR			
A1.1.3 Linear Inequalities			
Anchor Descriptor	Eligible Content		PA Core Standards
A1.1.3.1 Write, solve, and/or graph linear inequalities using various methods.	A1.1.3.1.1	Write or solve compound inequalities and/or graph their solution sets on a number line (may include absolute value inequalities).	CC.2.1.HS.F.5 Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.
	A1.1.3.1.2	Identify or graph the solution set to a linear inequality on a number line.	CC.2.2.HS.D.7 Create and graph equations or inequalities to describe numbers or relationships.
	A1.1.3.1.3	Interpret solutions to problems in the context of the problem situation. <u>Note:</u> Linear inequalities only.	CC.2.2.HS.D.9 Use reasoning to solve equations and justify the solution method.
			CC.2.2.HS.D.10 Represent, solve, and interpret equations/inequalities and systems of equations/inequalities algebraically and graphically.
Anchor Descriptor	Eligible Content		PA Core Standards
A1.1.3.2 Write, solve, and/or graph systems of linear inequalities using various methods.	A1.1.3.2.1	Write and/or solve a system of linear inequalities using graphing. <u>Note:</u> Limit systems to two linear inequalities.	CC.2.1.HS.F.5 Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.
	A1.1.3.2.2	Interpret solutions to problems in the context of the problem situation. <u>Note:</u> Limit systems to two linear inequalities.	CC.2.2.HS.D.7 Create and graph equations or inequalities to describe numbers or relationships.
			CC.2.2.HS.D.10 Represent, solve, and interpret equations/inequalities and systems of equations/inequalities algebraically and graphically.

Eligible Content may be assessed using problem-solving situations.

ASSESSMENT ANCHOR A1.2.1 Functions		
Anchor Descriptor	Eligible Content	PA Core Standards
A1.2.1.1 Analyze and/or use patterns or relations.	A1.2.1.1.1 Analyze a set of data for the existence of a pattern and represent the pattern algebraically and/or graphically.	CC.2.2.8.C.1 Define, evaluate, and compare functions.
	A1.2.1.1.2 Determine whether a relation is a function, given a set of points or a graph.	CC.2.2.8.C.2 Use concepts of functions to model relationships between quantities.
	A1.2.1.1.3 Identify the domain or range of a relation (may be presented as ordered pairs, a graph, or a table).	CC.2.2.HS.C.1 Use the concept and notation of functions to interpret and apply them in terms of their context.
		CC.2.2.HS.C.2 Graph and analyze functions and use their properties to make connections between the different representations. CC.2.2.HS.C.3 Write functions or sequences that model relationships between two quantities. CC.2.4.HS.B.2 Summarize, represent, and interpret data on two categorical and quantitative variables.

Eligible Content may be assessed using problem-solving situations.

Anchor Descriptor	Eligible Content	PA Core Standards
A1.2.1.2 Interpret and/or use linear functions and their equations, graphs, or tables.	A1.2.1.2.1 Create, interpret, and/or use the equation, graph, or table of a linear function.	CC.2.1.HS.F.3 Apply quantitative reasoning to choose and interpret units and scales in formulas, graphs, and data displays.
	A1.2.1.2.2 Translate from one representation of a linear function to another (i.e., graph, table, and equation).	CC.2.1.HS.F.4 Use units as a way to understand problems and to guide the solution of multi-step problems. CC.2.2.8.B.2 Understand the connections between proportional relationships, lines, and linear equations. CC.2.2.8.C.1 Define, evaluate, and compare functions. CC.2.2.8.C.2 Use concepts of functions to model relationships between quantities. CC.2.2.HS.C.2 Graph and analyze functions and use their properties to make connections between the different representations. CC.2.2.HS.C.3 Write functions or sequences that model relationships between two quantities. CC.2.2.HS.C.4 Interpret the effects transformations have on functions and find the inverses of functions. CC.2.2.HS.C.6 Interpret functions in terms of the situations they model. CC.2.4.HS.B.2 Summarize, represent, and interpret data on two categorical and quantitative variables.

Eligible Content may be assessed using problem-solving situations.

ASSESSMENT ANCHOR A1.2.2 Coordinate Geometry		
Anchor Descriptor	Eligible Content	PA Core Standards
A1.2.2.1 Describe, compute, and/or use the rate of change (slope) of a line.	A1.2.2.1.1 Identify, describe, and/or use constant rates of change.	CC.2.2.8.C.2 Use concepts of functions to model relationships between quantities.
	A1.2.2.1.2 Apply the concept of linear rate of change (slope) to solve problems.	CC.2.2.HS.C.1 Use the concept and notation of functions to interpret and apply them in terms of their context.
	A1.2.2.1.3 Write or identify a linear equation when given - the graph of the line, - two points on the line, or - the slope and a point on the line. <u>Note:</u> Linear equation may be in point-slope, standard, and/or slope-intercept form.	CC.2.2.HS.C.2 Graph and analyze functions and use their properties to make connections between the different representations.
	A1.2.2.1.4 Determine the slope and/or y-intercept represented by a linear equation or graph.	CC.2.2.HS.C.3 Write functions or sequences that model relationships between two quantities. CC.2.2.HS.C.5 Construct and compare linear, quadratic, and exponential models to solve problems. CC.2.2.HS.C.6 Interpret functions in terms of the situations they model. CC.2.4.HS.B.1 Summarize, represent, and interpret data on a single count or measurement variable.

Eligible Content may be assessed using problem-solving situations.

Anchor Descriptor	Eligible Content	PA Core Standards
A1.2.2.2 Analyze and/or interpret data on a scatter plot.	A1.2.2.2.1 Draw, identify, find, and/or write an equation for a line of best fit for a scatter plot.	CC.2.2.HS.C.6 Interpret functions in terms of the situations they model. CC.2.4.8.B.1 Analyze and/or interpret bivariate data displayed in multiple representations. CC.2.4.HS.B.2 Summarize, represent, and interpret data on two categorical and quantitative variables. CC.2.4.HS.B.3 Analyze linear models to make interpretations based on the data.

Eligible Content may be assessed using problem-solving situations.

ASSESSMENT ANCHOR A1.2.3 Data Analysis		
Anchor Descriptor	Eligible Content	PA Core Standards
A1.2.3.1 Use measures of dispersion to describe a set of data.	A1.2.3.1.1 Calculate and/or interpret the range, quartiles, and interquartile range of data.	CC.2.4.HS.B.1 Summarize, represent, and interpret data on a single count or measurement variable. CC.2.4.HS.B.3 Analyze linear models to make interpretations based on the data.
Anchor Descriptor	Eligible Content	PA Core Standards
A1.2.3.2 Use data displays in problem-solving settings and/or to make predictions.	A1.2.3.2.1 Estimate or calculate to make predictions based on a circle, line, bar graph, measure of central tendency, or other representation.	CC.2.4.HS.B.1 Summarize, represent, and interpret data on a single count or measurement variable.
	A1.2.3.2.2 Analyze data, make predictions, and/or answer questions based on displayed data (box-and-whisker plots, stem-and-leaf plots, scatter plots, measures of central tendency, or other representations).	CC.2.4.HS.B.3 Analyze linear models to make interpretations based on the data.
	A1.2.3.2.3 Make predictions using the equations or graphs of best-fit lines of scatter plots.	CC.2.4.HS.B.5 Make inferences and justify conclusions based on sample surveys, experiments, and observational studies.
Anchor Descriptor	Eligible Content	PA Core Standards
A1.2.3.3 Apply probability to practical situations.	A1.2.3.3.1 Find probabilities for compound events (e.g., find probability of red and blue, find probability of red or blue) and represent as a fraction, decimal, or percent.	CC.2.4.7.B.3 Investigate chance processes and develop, use, and evaluate probability models. CC.2.4.HS.B.4 Recognize and evaluate random processes underlying statistical experiments. CC.2.4.HS.B.7 Apply the rules of probability to compute probabilities of compound events in a uniform probability model.

Eligible Content may be assessed using problem-solving situations.

ALGEBRA I CONSTRUCTED-RESPONSE QUESTIONS

GENERAL DESCRIPTION OF SCORING GUIDELINES

4 Points

- The response demonstrates a *thorough* understanding of the mathematical concepts and procedures required by the task.
- The response provides correct answer(s) with clear and complete mathematical procedures shown and a correct explanation, as required by the task. Response may contain a minor “blemish” or omission in work or explanation that does not detract from demonstrating a *thorough* understanding.

3 Points

- The response demonstrates a *general* understanding of the mathematical concepts and procedures required by the task.
- The response and explanation (as required by the task) are mostly complete and correct. The response may have minor errors or omissions that do not detract from demonstrating a *general* understanding.

2 Points

- The response demonstrates a *partial* understanding of the mathematical concepts and procedures required by the task.
- The response is somewhat correct with *partial* understanding of the required mathematical concepts and/or procedures demonstrated and/or explained. The response may contain some work that is incomplete or unclear.

1 Point

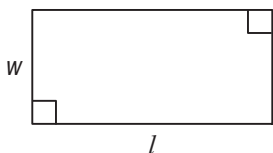
- The response demonstrates a *minimal* understanding of the mathematical concepts and procedures required by the task.

0 Points

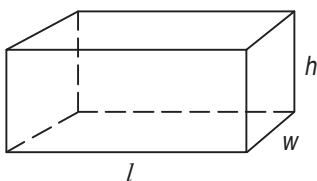
- The response has no correct answer and *insufficient* evidence to demonstrate any understanding of the mathematical concepts and procedures required by the task.

ALGEBRA I FORMULA SHEET

Formulas that you may need to solve questions on this exam are found below.
You may use calculator π or the number 3.14.



$$A = lw$$



$$V = lwh$$

Linear Equations

Slope: $m = \frac{y_2 - y_1}{x_2 - x_1}$

Point-Slope Formula: $(y - y_1) = m(x - x_1)$

Slope-Intercept Formula: $y = mx + b$

Standard Equation of a Line: $Ax + By = C$

Arithmetic Properties

Additive Inverse: $a + (-a) = 0$

Multiplicative Inverse: $a \cdot \frac{1}{a} = 1$

Commutative Property: $a + b = b + a$
 $a \cdot b = b \cdot a$

Associative Property: $(a + b) + c = a + (b + c)$
 $(a \cdot b) \cdot c = a \cdot (b \cdot c)$

Identity Property: $a + 0 = a$
 $a \cdot 1 = a$

Distributive Property: $a \cdot (b + c) = a \cdot b + a \cdot c$

Multiplicative Property of Zero: $a \cdot 0 = 0$

Additive Property of Equality:
If $a = b$, then $a + c = b + c$

Multiplicative Property of Equality:
If $a = b$, then $a \cdot c = b \cdot c$

Pennsylvania Common Core Standards
Mathematics

INTRODUCTION

The Pennsylvania Common Core Standards in Mathematics in grades PreK-5 lay a solid foundation in whole numbers, addition, subtraction, multiplication, division, fractions, and decimals. Taken together, these elements support a student’s ability to learn and apply more demanding math concepts and procedures. The middle school and high school standards call on students to practice applying mathematical ways of thinking to real world issues and challenges; they prepare students to think and reason mathematically. Additionally, they set a rigorous definition of college and career readiness by demanding that students develop a depth of understanding and ability to apply mathematics to novel situations, as college students and employees regularly do. Although the **standards** are **not a curriculum** or a prescribed series of activities, school entities will use them to develop a local school curriculum that will meet local students’ needs.

This document includes PA Common Core Standards for **Mathematics Content** and **Mathematical Practice**. The mathematics standards define what students should understand and be able to do. Mathematical Practice Standards describes the habits of mind required to reach a level of mathematical proficiency.

PA Common Core Standards Mathematical Content and Mathematical Practice		
Standards for Mathematical Content	Standards for Mathematical Practice	
2.1 Numbers and Operations A) Counting and Cardinality B) Number and Operations in Base Ten C) Number and Operations—Fractions D) Ratios and Proportional Relationships E) The Number System F) Number and Quantity	- Make sense of problems and persevere in solving them. - Reason abstractly and quantitatively. - Construct viable arguments and critique the reasoning of others. - Model with mathematics. - Use appropriate tools strategically. - Attend to precision. - Look for and make use of structure. - Look for and make sense of regularity in repeated reasoning.	
2.2 Algebraic Concepts A) Operations and Algebra Thinking B) Expressions & Equations C) Functions D) Algebra		
2.3 Geometry A) Geometry		
2.4 Measurement, Data and Probability A) Measurement and Data B) Statistics and Probability		