**PA Core Standards for Mathematics**

**Middle School Math Compacted Curriculum**

**Model Unit Overview**

In this model the compacting begins in the sixth grade.

* 6th Grade: 6th grade standards and some of 7th grade
* 7th Grade: Remaining 7th grade standards and some of 8th grade
* 8th Grade: Remaining 8th grade standards and PA Core Standards for Algebra 1

The purpose of this document is to give an overview of one possible way the standards for Compacting Model could be grouped into units. The units are not in a specific order nor are they numbered. The LEA should decide order and number of days per unit. Please note that some standards have been intentionally placed in multiple units.

The **Standards for Mathematical Practice** should be infused throughout all units:

1. Make sense of problems and persevere in solving them.

2. Reason abstractly and quantitatively.

3. Construct viable arguments and critique the reasoning of others.

4. Model with mathematics.

5. Use appropriate tools strategically.

6. Attend to precision.

7. Look for and make use of structure.

8. Look for and express regularity in repeated reasoning

| **6th Grade** | | |  |
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| **Concepts** | **Competencies** | **Standards** | **Eligible Content** |
| Understanding Ratios and Rates | Represent ratio relationships in various forms.  Determine unit rates in context.    Convert measurement units using equivalent ratios.    Solve problems using ratio and rate reasoning. | CC.2.1.6.D.1 Understand ratio concepts and use ratio reasoning to solve problems | A-R.1.1.1 Use ratio language and notation (such as 3 to 4, 3:4, ) to describe a ratio relationship between two quantities.  A-R.1.1.2 Find the unit rate associated with a ratio *a*:*b* (with *b* ≠0) and use rate language in the context of a ratio relationship.  A-R.1.1.3 Construct tables of equivalent ratios relating quantities with whole-number measurements, find missing values in the tables, and/or plot the pairs of values on the coordinate plane. Use tables to compare ratios.  A-R.1.1.4 Solve unit rate problems including those involving unit pricing and constant speed. |
| Represent and analyze quantitative relationship between dependent and independent variables. | CC.2.2.6.B.3  Represent and analyze quantitative relationships between dependent and independent variables. | B-E.3.1.1 Write an equation to express the relationship between the dependent and independent variables.  B-E.3.1.2 Analyze the relationship between the dependent and independent variables using graphs and tables and/or relate these to an equation. |
| Compute unit rates associated with ratios of fractions.  Recognize and represent proportional relationships between quantities. | CC.2.1.7.D.1Analyze proportional relationships and use them to model and solve real-world and mathematical problems | [M07.A-R.1.1.1](http://www.pdesas.org/kmmauro/2016/8/23/660538/file.aspx)Compute unit rates associated with ratios of fractions, including ratios of lengths, areas, and other quantities measured in like or different units.  Example: If a person walks 1/2 mile in each 1/4 hour, compute the unit rate as the complex fraction 1/2 / 1/4 miles per hour, equivalently 2 miles per hour |
| Reasoning Proportionally with Percents | Solve problems using ratio and rate reasoning. | CC.2.1.6.D.1 Understand ratio concepts and use ratio reasoning to solve problems. | A-R.1.1.5 Find a percent of a quantity as a rate per 100 (e.g., 30% of a quantity means times the quantity); solve problems involving finding the whole, given a part and the percentage. |
| Multi-digit Computation and Finding Common Factors | Compute fluently with multi-digit numbers and find common factors and multiples.  Interpret and compute quotients of fraction.  Solve problems and compute fluently with whole numbers and decimals.  Find common multiples and factors including greatest common factor and least common multiple. | CC.2.1.6.E.1Apply and extend previous understandings of multiplication and division to divide fractions by fractions.  CC.2.1.6.E.2  Identify and choose appropriate processes to compute fluently with multi-digit numbers.  CC.2.1.6.E.3Develop and/or apply number theory concepts to find common factors and multiples. | A-N.2.1.1 Solve problems involving operations (+, –, ×, and ÷) with whole numbers, decimal (through thousandths), straight computation, or word problems.  A-N.2.2.1 Find the greatest common factor of two whole numbers less than or equal to 100 and the least common multiple of two whole numbers less than or equal to 12. |
| Apply the properties of operations to generate equivalent expressions | CC.2.2.6.B.1Apply and extend previous understandings of arithmetic to algebraic expressions. | B-E.1.1.5 Apply the properties of operations to generate equivalent expressions. |
| Dividing Fractions | Interpret and compute quotients of fraction. | CC.2.1.6.E.1Apply and extend previous understandings of multiplication and division to divide fractions by fractions. | A-N.1.1.1 Interpret and compute quotients of fractions (including mixed numbers), and solve word problems involving division of fractions by fractions. |
| Extending the Number System | Use the distributive property to express a sum of two numbers.  Use positive and negative numbers to represent quantities in real world contexts.  Plot integers and other rational numbers on a number line and on a coordinate graph.  Interpret the opposite and absolute value of an integer as its distance from zero on a number line.  Compare and order rational numbers. | CC.2.1.6.E.4Apply and extend previous understandings of numbers to the system of rational numbers. | A-N.3.1.1 Represent quantities in real-world contexts using positive and negative numbers, explaining the meaning of 0 in each situation (e.g., temperature above/below zero, elevation above/below sea level, credits/debits, positive/negative electric charge).  A-N.3.1.2 Determine the opposite of a number and recognize that the opposite of the opposite of a number is the number itself (e.g., –(–3) = 3; 0 is its own opposite).  A-N.3.1.3 Locate and plot integers and other rational numbers on a horizontal or vertical number line; locate and plot pairs of integers and other rational numbers on a coordinate plane. |
| Relationships in the Coordinate Plane | Use the distributive property to express a sum of two numbers.  Use positive and negative numbers to represent quantities in real world contexts.  Plot integers and other rational numbers on a number line and on a coordinate graph.  Interpret the opposite and absolute value of an integer as its distance from zero on a number line. | CC.2.1.6.E.4Apply and extend previous understandings of numbers to the system of rational numbers. | A-N.3.1.3 Locate and plot integers and other rational numbers on a horizontal or vertical number line; locate and plot pairs of integers and other rational numbers on a coordinate plane.  A-N.3.2.3 Solve real-world and mathematical problems by plotting points in all four quadrants of the coordinate plane. Include use of coordinates and absolute value to find distance between points with the same first coordinate or the same second coordinate. |
| Determine the area of triangles, quadrilaterals, irregular polygons and compound polygons  Calculate the area of a polygon on a plane given the coordinates of the vertices  Find volumes of right rectangular prisms with fractional edge lengths  Use nets to find surface area of 3 – dimensional figures | CC.2.3.6.A.1  Apply appropriate tools to solve real-world and mathematical problems involving area, surface area, and volume. | C-G.1.1.4 Given coordinates for the vertices of a polygon in the plane, use the coordinates to find side lengths and area of the polygon (limited to triangles and special quadrilaterals). Formulas will be provided. |
| Positive and Negative Rational Numbers | Solve real-world and mathematical problems involving the four operations with rational numbers. | CC.2.1.7.E.1  Apply and extend previous understandings of operations with fractions to operations with rational numbers. | M07.A-N.1.1.1 Apply properties of operations to add and subtract rational numbers, including real-world contexts.  M07.A-N.1.1.2 Represent addition and subtraction on a horizontal or vertical number line.  M07.A-N.1.1.3 Apply properties of operations to multiply and divide rational numbers, including real-world contexts; demonstrate that the decimal form of a rational number terminates or eventually repeats. |
| Algebraic Expressions | Write, identify and evaluate numerical expressions involving exponents.  Write, read and evaluate algebraic expressions  Apply the properties of operations to generate equivalent expressions | CC.2.2.6.B.1Apply and extend previous understandings of arithmetic to algebraic expressions. | B-E.1.1.1 Write and evaluate numerical expressions involving whole-number exponents.  B-E.1.1.2 Write algebraic expressions from verbal descriptions.  B-E.1.1.3 Identify parts of an expression using mathematical terms (e.g., sum, term, product, factor, quotient, coefficient, quantity).  B-E.1.1.4 Evaluate expressions at specific values of their variables, including expressions that arise from formulas used in real-world problems.  B-E.1.1.5 Apply the properties of operations to generate equivalent expressions. |
| Equations and Inequalities | Reason about and solve one-variable equations and inequalities.  Solve and interpret one variable equations or inequalities in real world and mathematical problems | CC.2.2.6.B.2  Understand the process of solving a one-variable equation or inequality and apply it to real-world and mathematical problems. | B-E.2.1.1 Use substitution to determine whether a given number in a specified set makes an equation or inequality true.  B-E.2.1.2 Write algebraic expressions to represent real-world or mathematical problems.  B-E.2.1.3 Solve real-world and mathematical problems by writing and solving equations of the form *x* + *p* = *q* and *px* = *q* for cases in which *p*, *q*, and *x* are all non-negative rational numbers.  B-E.2.1.4 Write an inequality of the form *x* > *c* or *x* < *c* to represent a constraint or condition in a real-world or mathematical problem and/or represent solutions of such inequalities on number lines. |
| Problem Solving with Area in 2-Dimensional Shapes | Determine the area of triangles, quadrilaterals, irregular polygons and compound polygons  Calculate the area of a polygon on a plane given the coordinates of the vertices  Find volumes of right rectangular prisms with fractional edge lengths  Use nets to find surface area of 3 – dimensional figures | CC.2.3.6.A.1  Apply appropriate tools to solve real-world and mathematical problems involving area, surface area, and volume. | C-G.1.1.1 Determine the area of triangles and special quadrilaterals (i.e., square, rectangle, parallelogram, rhombus, and trapezoid). Formulas will be provided.  C-G.1.1.2 Determine the area of irregular or compound polygons.  C-G.1.1.4 Given coordinates for the vertices of a polygon in the plane, use the coordinates to find side lengths and area of the polygon (limited to triangles and special quadrilaterals). Formulas will be provided. |
| Use properties of angle types and properties of angles formed when two parallel lines are cut by a transversal line to solve problems.  Solve problems involving area and circumference of a circle(s).  Solve mathematical problems involving area, volume and surface area of two- and three-dimensional objects. | CC.2.3.7.A.1Visualize and represent geometric figures and describe the relationships between them. | M07.C-G.1.1.4Describe the two-dimensional figures that result from slicing three-dimensional figures. Example: Describe plane sections of right rectangular prisms and right rectangular pyramids. |
| Problem Solving with Volume and Surface Area | Find volumes of right rectangular prisms with fractional edge lengths  Use nets to find surface area of 3 – dimensional figures | CC.2.3.6.A.1  Apply appropriate tools to solve real-world and mathematical problems involving area, surface area, and volume. | C-G.1.1.3 Determine the volume of right rectangular prisms with fractional edge lengths. Formulas will be provided.  C-G.1.1.5 Represent three-dimensional figures using nets made of rectangles and triangles.  C-G.1.1.6 Determine the surface area of triangular and rectangular prisms (including cubes). Formulas will be provided. |
| Statistical Investigations and Sampling | Display data in dot plots, histograms and box-and-whisker plots  Determine quantitative measures of center and variability | CC.2.4.6.B.1  Demonstrate an understanding of statistical variability by displaying, analyzing, and summarizing distributions. | D-S.1.1.1 Display numerical data in plots on a number line, including line plots, histograms, and box-and- whisker plots.  D-S.1.1.2 Determine quantitative measures of center (e.g., median, mean, mode) and variability (e.g., range, interquartile range, mean absolute deviation). |
| Draw inferences about two populations based on random sampling concepts.  Determine and approximate relative frequencies and probabilities of events.  Find the probability of a simple event, including the probability of a simple event not occurring. | CC.2.4.7.B.1  Draw informal comparative inferences about two populations. | M07.D-S.1.1.1 Determine whether a sample is a random sample given a real-world situation.  M07.D-S.1.1.2 Use data from a random sample to draw inferences about a population with an unknown characteristic of interest. Example 1: Estimate the mean word length in a book by randomly sampling words from the book. Example 2: Predict the winner of a school election based on randomly sampled survey data. |
| Analyzing Data | Choose the appropriate measure of center and variability for a set of data | CC.2.4.6.B.1  Demonstrate an understanding of statistical variability by displaying, analyzing, and summarizing distributions | D-S.1.1.3 Describe any overall pattern and any deviations from the overall pattern with reference to the context in which the data were gathered.  D-S.1.1.4 Relate the choice of measures of center and variability to the shape of the data distribution and the context in which the data were gathered. |
| Draw inferences about two populations based on random sampling concepts.  Determine and approximate relative frequencies and probabilities of events.  Find the probability of a simple event, including the probability of a simple event not occurring. | CC.2.4.7.B.2  Draw inferences about populations based on random sampling concepts. | M07.D-S.2.1.1Use data from a random sample to draw inferences about a population with an unknown characteristic of interest.  Example 1: Estimate the mean word length in a book by randomly sampling words from the book. Example 2: Predict the winner of a school election based on randomly sampled survey data. |

| **7th Grade** | | |  |
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| **Concepts** | **Competences** | **Standards** | **Eligible Content** |
| Proportional Reasoning and Relationships | Compute unit rates associated with ratios of fractions.  Recognize and represent proportional relationships between quantities.  Use proportional relationships to solve multistep ratio and percent problems. | CC.2.1.7.D.1  Analyze proportional relationships and use them to model and solve real-world and mathematical problems | M07.A-R.1.1.1 Compute unit rates associated with ratios of fractions, including ratios of lengths, areas, and other quantities measured in like or different units. Example: If a person walks 1/2 mile in each 1/4 hour, compute the unit rate as the complex fraction 1/2 / 1/4 miles per hour, equivalently 2 miles per hour.  M07.A-R.1.1.2 Determine whether two quantities are proportionally related (e.g., by testing for equivalent ratios in a table, graphing on a coordinate plane and observing whether the graph is a straight line through the origin).  M07.A-R.1.1.3 Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships.  M07.A-R.1.1.4 Represent proportional relationships by equations. Example: If total cost t is proportional to the number n of items purchased at a constant price p, the relationship between the total cost and the number of items can be expressed as t = pn. M07.A-R.1.1.5 Explain what a point (x, y) on the graph of a proportional relationship means in terms of the situation, with special attention to the points (0, 0) and (1, r), where r is the unit rate. |
| Solve problems involving scale drawings of geometric figures.  Apply the properties of all types of triangles based on angle and side measure including the triangle inequality theorem.  Describe the two-dimensional figures that result from slicing three-dimensional figures. | CC.2.3.7.A.2  Visualize and represent geometric figures and describe the relationships between them | M07.C-G.1.1.1 Solve problems involving scale drawings of geometric figures, including finding length and area.  M07.C-G.1.1.4 Describe the two-dimensional figures that result from slicing three-dimensional figures. Example: Describe plane sections of right rectangular prisms and right rectangular pyramids. |
| Analyze and describe linear relationships between two variables, using slope.  Make connections between slope, lines and linear equations.  Analyze, model and solve linear equations.  Analyze and solve pairs of simultaneous equations.  Interpret solutions to a linear equation and systems of two linear equations. | CC.2.2.8.B.2  Understand the connections between proportional relationships, lines, and linear equations. | M08.B-E.2.1.1 Graph proportional relationships, interpreting the unit rate as the slope of the graph. Compare two different proportional relationships represented in different ways. Example: Compare a distance-time graph to a distance-time equation to determine which of two moving objects has greater speed.  M08.B-E.2.1.2 Use similar right triangles to show and explain why the slope m is the same between any two distinct points on a non-vertical line in the coordinate plane.  M08.B-E.2.1.3 Derive the equation y = mx for a line through the origin and the equation y = mx + b for a line intercepting the vertical axis at b |
| Proportional Reasoning with Percents | Use proportional relationships to solve multistep ratio and percent problems. | CC.2.1.7.D.1  Analyze proportional relationships and use them to model and solve real-world and mathematical problems | M07.A-R.1.1.6Use proportional relationships to solve multi-step ratio and percent problems. Examples: simple interest, tax, markups and markdowns, gratuities and commissions, fees, percent increase and decrease. |
| Apply properties of operations to generate equivalent expressions.  Model and solve real world and mathematical problems using multiple representations such as algebraic, graphical and using tables | CC.2.2.7.B.3  Model and solve real-world and mathematical problems by using and connecting numerical, algebraic, and/or graphical representations. | M07.B-E.2.1.1 Apply properties of operations to calculate with numbers in any form; convert between forms as appropriate. Example: If a woman making $25 an hour gets a 10% raise, she will make an additional 1/10 of her salary an hour, or $2.50, for a new salary of $27.50 an hour (or 1.1 × $25 = $27.50). |
| Solving Inequalities | Apply properties of operations to generate equivalent expressions.  Solve multi-step equations or inequalities with one variable | CC.2.2.7.B.1  Apply properties of operations to generate equivalent expressions. | M07.B-E.1.1.1 Apply properties of operations to add, subtract, factor, and expand linear expressions with rational coefficients.  Example 1: The expression 1/2 • (x + 6) is equivalent to 1/2 • x + 3. Example 2: The expression 5.3 – y + 4.2 is equivalent to 9.5 – y (or – y + 9.5). Example 3: The expression 4w – 10 is equivalent to 2(2w – 5). |
| Solve multi-step equations or inequalities with one variable.  Solve and interpret multi-step real life and mathematical problems posed with positive and negative rational numbers. | CC.2.2.7.B.3  Model and solve real-world and mathematical problems by using and connecting numerical, algebraic, and/or graphical representations. | M07.B-E.2.2.2 Solve word problems leading to inequalities of the form px + q > r or px + q < r, where p, q, and r are specific rational numbers, and graph the solution set of the inequality.  Example: A salesperson is paid $50 per week plus $3 per sale. This week she wants her pay to be at least $100. Write an inequality for the number of sales the salesperson needs to make and describe the solutions. |
| Solving Equations | Use properties of operations to generate equivalent expressions. | CC.2.2.7.B.1Apply properties of operations to generate equivalent expressions. | M07.B-E.1.1.1 Apply properties of operations to add, subtract, factor, and expand linear expressions with rational coefficients. Example 1: The expression 1/2 • (x + 6) is equivalent to 1/2 • x + 3. Example 2: The expression 5.3 – y + 4.2 is equivalent to 9.5 – y (or – y + 9.5). Example 3: The expression 4w – 10 is equivalent to 2(2w – 5). |
| Solve real-life and mathematical problems using numerical and algebraic expressions and equations. | CC.2.2.7.B.3Model and solve realworld and mathematical problems by using and connecting numerical, algebraic, and/or graphical representations. | M07.B-E.2.2.1 Solve word problems leading to equations of the form px + q = r and p(x + q) = r, where p, q, and r are specific rational numbers. Example: The perimeter of a rectangle is 54 cm. Its length is 6 cm. What is its width?  M07.B-E.2.3.1 Determine the reasonableness of answer(s) or interpret the solution(s) in the context of the problem. Example: If you want to place a towel bar that is 9 3/4 inches long in the center of a door that is 27 1/2 inches wide, you will need to place the bar about 9 inches from each edge; this estimate can be used as a check on the exact computation. |
| Use properties of angle types and properties of angles formed when two parallel lines are cut by a transversal line to solve problems. | CC.2.3.7.A.1  Solve real-world and mathematical problems involving angle measure, area, surface area, circumference, and volume. | M07.C-G.2.1.1 Identify and use properties of supplementary, complementary, and adjacent angles in a multistep problem to write and solve simple equations for an unknown angle in a figure.  M07.C-G.2.1.2 Identify and use properties of angles formed when two parallel lines are cut by a transversal (e.g., angles may include alternate interior, alternate exterior, vertical, corresponding). |
| Analyze, model and solve linear equations.  Analyze and solve pairs of simultaneous equations.  Interpret solutions to a linear equation and systems of two linear equations. | CC.2.2.8.B.3  Analyze and solve linear equations and pairs of simultaneous linear equations. | M08.B-E.3.1.1 Write and identify linear equations in one variable with one solution, infinitely many solutions, or no solutions. Show which of these possibilities is the case by successively transforming the given equation into simpler forms until an equivalent equation of the form x = a, a = a, or a = b results (where a and b are different numbers).  M08.B-E.3.1.2 Solve linear equations that have rational number coefficients, including equations whose solutions require expanding expressions using the distributive property and collecting like terms. |
| Area and Volume | Solve problems involving area and circumference of a circle(s).  Solve mathematical problems involving area, volume and surface area of two- and three-dimensional objects. | CC.2.3.7.A.1  Solve real-world and mathematical problems involving angle measure, area, surface area, circumference, and volume. | M07.C-G.2.2.1 Find the area and circumference of a circle. Solve problems involving area and circumference of a circle(s). Formulas will be provided.  M07.C-G.2.2.2 Solve real-world and mathematical problems involving area, volume, and surface area of two-and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms. Formulas will be provided. |
| Apply concepts of volume of cylinders, cones and spheres to solve real-world and mathematical problems. | CC.2.3.8.A.1  Apply the concepts of volume of cylinders, cones, and spheres to solve real-world and mathematical problems. | M08.C-G.3.1.1Apply formulas for the volumes of cones, cylinders, and spheres to solve real-world and mathematical problems. Formulas will be provided. |
| Work with radicals and integer exponents.  Apply concepts of integer exponents to generate equivalent expressions.  Use and evaluate square roots and cube roots to represent solutions to equations. | CC.2.2.8.B.1  Apply concepts of radicals and integer exponents to generate equivalent expressions. | M08.B-E.1.1.2Use square root and cube root symbols to represent solutions to equations of the form x2 = p and x3 = p, where p is a positive rational number. Evaluate square roots of perfect squares (up to and including 122 ) and cube roots of perfect cubes (up to and including 53 ) without a calculator. Example: If x2 = 25 then x = ±√25. |
| Pythagorean Theorem | Know that there are numbers that are not rational, and approximate them by rational numbers.  Convert a terminating or repeating decimal into a rational number.  Use rational approximations of irrational numbers to compare the size of irrational numbers. | CC.2.1.8.E.1  Distinguish between rational and irrational numbers using their properties.  CC.2.1.8.E.4  Estimate irrational numbers by comparing them to rational numbers. | M08.A-N.1.1.1 Determine whether a number is rational or irrational. For rational numbers, show that the decimal expansion terminates or repeats (limit repeating decimals to thousandths).  M08.A-N.1.1.2 Convert a terminating or repeating decimal to a rational number (limit repeating decimals to thousandths).  M08.A-N.1.1.3 Estimate the value of irrational numbers without a calculator (limit whole number radicand to less than 144). Example: √5 is between 2 and 3 but closer to 2.  M08.A-N.1.1.4 Use rational approximations of irrational numbers to compare and order irrational numbers.  M08.A-N.1.1.5 Locate/identify rational and irrational numbers at their approximate locations on a number line. |
| Apply the Pythagorean Theorem and its converse to solve mathematical problems in two and three dimensions. | CC.2.3.8.A.3  Understand and apply the Pythagorean Theorem to solve problems. | M08.C-G.2.1.1 Apply the converse of the Pythagorean theorem to show a triangle is a right triangle.  M08.C-G.2.1.2 Apply the Pythagorean theorem to determine unknown side lengths in right triangles in real-world and mathematical problems in two and three dimensions. (Figures provided for problems in three dimensions will be consistent with Eligible Content in grade 8 and below.)  M08.C-G.2.1.3 Apply the Pythagorean theorem to find the distance between two points in a coordinate system. |
| Use coordinates to prove simple geometric theorems algebraically. | CC.2.3.HS.A.11  Apply coordinate geometry to prove simple geometric theorems algebraically. |  |
| Probability of Simple Events | Draw informal comparative inferences about two populations using measures of center and measures of variability.  Draw inferences about two populations based on random sampling concepts.  Find probabilities of independent compound events.  Predict the approximate relative frequency given the probability. | CC.2.4.7.B.2  Draw informal comparative inferences about two populations.  CC.2.4.7.B.3 Investigate chance processes and develop, use, and evaluate probability models. | M07.D-S.2.1.1Compare two numerical data distributions using measures of center and variability. Example 1: The mean height of players on the basketball team is 10 cm greater than the mean height of players on the soccer team. This difference is equal to approximately twice the variability (mean absolute deviation) on either team. On a line plot, note the difference between the two distributions of heights.  Example 2: Decide whether the words in a chapter of a seventh-grade science book are generally longer than the words in a chapter of a fourthgrade science book  M07.D-S.3.1.1Predict or determine whether some outcomes are certain, more likely, less likely, equally likely, or impossible (i.e., a probability near 0 indicates an unlikely event, a probability around 1/2 indicates an event that is neither unlikely nor likely, and a probability near 1 indicates a likely event).  M07.D-S.3.2.1Determine the probability of a chance event given relative frequency. Predict the approximate relative frequency given the probability. Example: When rolling a number cube 600 times, predict that a 3 or 6 would be rolled roughly 200 times but probably not exactly 200 times.  M07.D-S.3.2.2 Find the probability of a simple event, including the probability of a simple event not occurring. Example: What is the probability of not rolling a 1 on a number cube? |
| Probability of Compound Events | Find probabilities of independent compound events.  Predict the approximate relative frequency given the probability. | CC.2.4.7.B.3 Investigate chance processes and develop, use, and evaluate probability models. | M07.D-S.3.2.3 Find probabilities of independent compound events using organized lists, tables, tree diagrams, and simulation. |
| Relationships in Geometric Figures | Solve mathematical problems involving area, volume and surface area of two- and three-dimensional objects. | CC.2.3.7.A.1  Solve real-world and mathematical problems involving angle measure, area, surface area, circumference, and volume. | M07.C-G.2.2.2 Solve real-world and mathematical problems involving area, volume, and surface area of two-and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms. Formulas will be provided. |
| Understand congruence and similarity using physical models, transparencies, or geometry software.  Apply concepts of volume of cylinders, cones and spheres to solve real-world and mathematical problems.  Use transformations to demonstrate congruence and similarity of geometric figures. | CC.2.3.8.A.2  Understand and apply congruence, similarity, and geometric transformations using various tools. | M08.C-G.1.1.1 Identify and apply properties of rotations, reflections, and translations. Example: Angle measures are preserved in rotations, reflections, and translations.  M08.C-G.1.1.2 Given two congruent figures, describe a sequence of transformations that exhibits the congruence between them.  M08.C-G.1.1.3 Describe the effect of dilations, translations, rotations, and reflections on two-dimensional figures using coordinates.  M08.C-G.1.1.4 Given two similar two-dimensional figures, describe a sequence of transformations that exhibits the similarity between them. |
| Analyze, model and solve linear equations. | CC.2.2.8.B.3  Analyze and solve linear equations and pairs of simultaneous linear equations. | M08.B-E.3.1.1 Write and identify linear equations in one variable with one solution, infinitely many solutions, or no solutions. Show which of these possibilities is the case by successively transforming the given equation into simpler forms until an equivalent equation of the form x = a, a = a, or a = b results (where a and b are different numbers).  M08.B-E.3.1.2 Solve linear equations that have rational number coefficients, including equations whose solutions require expanding expressions using the distributive property and collecting like terms. |
| Understanding Similarity through Transformations | Draw, construct, and describe geometrical figures and describe the relationships between them.  Solve problems involving scale drawings of geometric figures.  Apply the properties of all types of triangles based on angle and side measure including the triangle inequality theorem.  Describe the two-dimensional figures that result from slicing three-dimensional figures. | CC.2.3.7.A.2Visualize and represent geometric figures and describe the relationships between them. | M07.C-G.1.1.1 Solve problems involving scale drawings of geometric figures, including finding length and area.  M07.C-G.1.1.2 Identify or describe the properties of all types of triangles based on angle and side measures.  M07.C-G.1.1.3 Use and apply the triangle inequality theorem.  M07.C-G.1.1.4 Describe the two-dimensional figures that result from slicing three-dimensional figures. Example: Describe plane sections of right rectangular prisms and right rectangular pyramids. |
| Use transformations to demonstrate congruence and similarity of geometric figures.  Use various tools to understand and apply geometric transformations to geometric figures. | CC.2.3.8.A.2  Understand and apply congruence, similarity, and geometric transformations using various tools. | M08.C-G.1.1.3 Describe the effect of dilations, translations, rotations, and reflections on two-dimensional figures using coordinates.  M08.C-G.1.1.4 Given two similar two-dimensional figures, describe a sequence of transformations that exhibits the similarity between them. |
| Understanding Congruence through Transformations | Draw, construct, and describe geometrical figures and describe the relationships between them. | CC.2.3.7.A.2Visualize and represent geometric figures and describe the relationships between them. | M07.C-G.1.1.2 Identify or describe the properties of all types of triangles based on angle and side measures.  M07.C-G.1.1.3 Use and apply the triangle inequality theorem.  M07.C-G.1.1.4 Describe the two-dimensional figures that result from slicing three-dimensional figures. Example: Describe plane sections of right rectangular prisms and right rectangular pyramids. |
| Understand congruence and similarity using physical models, transparencies, or geometry software.  Use transformations to demonstrate congruence and similarity of geometric figures.  Use various tools to understand and apply geometric transformations to geometric figures. | CC.2.3.8.A.2  Understand and apply congruence, similarity, and geometric transformations using various tools. | M08.C-G.1.1.2 Given two congruent figures, describe a sequence of transformations that exhibits the congruence between them.  M08.C-G.1.1.3 Describe the effect of dilations, translations, rotations, and reflections on two-dimensional figures using coordinates. |

| **8th Grade (Algebra 1)** | | |  |
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| **Concepts** | **Competences** | **Standards** | **Eligible Content** |
| Working with Exponents | Work with radicals and integer exponents.  Use and evaluate square roots and cube roots to represent solutions to equations. | CC.2.2.8.B.1Apply concepts of radicals and integer exponents to generate equivalent expressions. | M08.B-E.1.1.1Apply one or more properties of integer exponents to generate equivalent numerical expressions without a calculator (with final answers expressed in exponential form with positive exponents). Properties will be provided. Example: 312 × 3 ⎯15 = 3 ⎯3 = 1/(33 )  M08.B-E.1.1.3 Estimate very large or very small quantities by using numbers expressed in the form of a single digit times an integer power of 10 and express how many times larger or smaller one number is than another. Example: Estimate the population of the United States as 3 × 108 and the population of the world as 7 × 109 and determine that the world population is more than 20 times larger than the United States’ population.  M08.B-E.1.1.4 Perform operations with numbers expressed in scientific notation, including problems where both decimal and scientific notation are used. Express answers in scientific notation and choose units of appropriate size for measurements of very large or very small quantities (e.g., use millimeters per year for seafloor spreading). Interpret scientific notation that has been generated by technology (e.g., interpret 4.7EE9 displayed on a calculator as 4.7 × 109 ). |
| Extend the properties of exponents to rational exponents. | CC.2.1.HS.F.1  Apply and extend the properties of exponents to solve problems with rational exponents. | A1.1.1.1.1, A1.1.1.1.2, A1.1.1.3.1  N.RN.  1. Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents. For example, we define 51/3 to be the cube root of 5 because we want (51/3 ) 3 = 5(1/3)3 to hold, so (51/3 ) 3 must equal 5.  2. Rewrite expressions involving radicals and rational exponents using the properties of exponents. |
| One-Variable Equations and Inequalities | Create equations that describe numbers or relationships. | CC.2.2.HS.D.7  Create and graph equations or inequalities to describe numbers or relationships. | A1.1.2.1.1, A1.1.2.1.2, A1.1.2.1.3, A1.1.2.2.1, A1.1.2.2.2, A1.1.3.1.1, A1.1.3.1.2, A1.1.3.1.3, A1.1.3.2.1, A1.1.3.2.2  A.CED.  1. Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear and quadratic functions, and simple rational and exponential functions.  2. Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.  3. Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context. For example, represent inequalities describing nutritional and cost constraints on combinations of different foods.  4. Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. For example, rearrange Ohm’s law V = IR to highlight resistance R. Linear, quadratic, and exponential (integer inputs only); for A.CED.3 linear only. |
| Understand solving equations as a process of reasoning and explain the reasoning. | CC.2.2.HS.D.8  Apply inverse operations to solve equations or formulas for a given variable. | A1.1.2.1.1, A1.1.2.1.2, A1.1.2.1.3  A.REI.  1. Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.  2. Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise. |
| Solve equations and inequalities in one variable. | CC.2.2.HS.D.10  Represent, solve, and interpret equations/inequalities and systems of equations/inequalities algebraically and graphically. | A1.1.2.1.1, A1.1.2.1.2, A1.1.2.1.3, A1.1.2.2.1, A1.1.2.2.2, A1.1.3.1.1, A1.1.3.1.2, A1.1.3.1.3, A1.1.3.2.1, A1.1.3.2.2  A.REI.  3. Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.  4. Solve quadratic equations in one variable. a. Use the method of completing the square to transform any quadratic equation in x into an equation of the form (x – p) 2 = q that has the same solutions. Derive the quadratic formula from this form. b. Solve quadratic equations by inspection (e.g., for x 2 = 49), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as a ± bi for real numbers a and b.  5. Prove that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions.  6. Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.  7. Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically. For example, find the points of intersection between the line y = –3x and the circle x 2 + y 2 = 3.  10. Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).  11. Explain why the x-coordinates of the points where the graphs of the equations y = f(x) and y = g(x) intersect are the solutions of the equation f(x) = g(x); find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where f(x) and/or g(x) are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.  12. Graph the solutions to a linear inequality in two variables as a half-plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes. |
| Systems of Equations and Inequalities | Analyze, model and solve linear equations.  Analyze and solve pairs of simultaneous equations.  Use and evaluate square roots and cube roots to represent solutions to equations | CC.2.2.8.B.3 Analyze and solve linear equations and pairs of simultaneous linear equations. | M08.B-E.3.1.3 Interpret solutions to a system of two linear equations in two variables as points of intersection of their graphs because points of intersection satisfy both equations simultaneously.  M08.B-E.3.1.4 Solve systems of two linear equations in two variables algebraically and estimate solutions by graphing the equations. Solve simple cases by inspection. Example: 3x + 2y = 5 and 3x + 2y = 6 have no solution because 3x + 2y cannot simultaneously be 5 and 6.  M08.B-E.3.1.5 Solve real-world and mathematical problems leading to two linear equations in two variables. Example: Given coordinates for two pairs of points, determine whether the line through the first pair of points intersects the line through the second pair. |
| Create equations that describe numbers or relationships | CC.2.2.HS.D.7  Create and graph equations or inequalities to describe numbers or relationships. | A1.1.2.1.1, A1.1.2.1.2, A1.1.2.1.3, A1.1.2.2.1, A1.1.2.2.2, A1.1.3.1.1, A1.1.3.1.2, A1.1.3.1.3, A1.1.3.2.1, A1.1.3.2.2  A.CED.  1. Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear and quadratic functions, and simple rational and exponential functions. 2. Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.  3. Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context. For example, represent inequalities describing nutritional and cost constraints on combinations of different foods.  4. Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. For example, rearrange Ohm’s law V = IR to highlight resistance R. Linear, quadratic, and exponential (integer inputs only); for A.CED.3 linear only. |
| Solve systems of equations.  Represent and solve equations and inequalities graphically. | CC.2.2.HS.D.10  Represent, solve, and interpret equations/inequalities and systems of equations/inequalities algebraically and graphically. | A1.1.2.1.1, A1.1.2.1.2, A1.1.2.1.3, A1.1.2.2.1, A1.1.2.2.2, A1.1.3.1.1, A1.1.3.1.2, A1.1.3.1.3, A1.1.3.2.1, A1.1.3.2.2  A.REI.  3. Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.  4. Solve quadratic equations in one variable. a. Use the method of completing the square to transform any quadratic equation in x into an equation of the form (x – p) 2 = q that has the same solutions. Derive the quadratic formula from this form. b. Solve quadratic equations by inspection (e.g., for x 2 = 49), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as a ± bi for real numbers a and b.  5. Prove that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions.  6. Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.  7. Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically. For example, find the points of intersection between the line y = –3x and the circle x 2 + y 2 = 3.  10. Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).  11. Explain why the x-coordinates of the points where the graphs of the equations y = f(x) and y = g(x) intersect are the solutions of the equation f(x) = g(x); find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where f(x) and/or g(x) are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.  12. Graph the solutions to a linear inequality in two variables as a half-plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes. |
| Area and Volume | Explain volume formulas and use them to solve problems.  Reason quantitatively and use units to solve problems. | CC.2.3.HS.A.12  Explain volume formulas and use them to solve problems.  . | G.2.3.1.1, G.2.3.1.2, G.2.3.1.3 |
| Function Introduction | Define, evaluate, and compare functions.  Define, interpret, and compare functions displayed algebraically, graphically, numerically in tables, or by verbal descriptions. | CC.2.2.8.C.1  Define, evaluate, and compare functions. | M08.B-F.1.1.1 Determine whether a relation is a function. |
| Use functions to model relationships between quantities.  Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values. | CC.2.2.8.C.2  Use concepts of functions to model relationships between quantities. | M08.B-F.2.1.2 Describe qualitatively the functional relationship between two quantities by analyzing a graph (e.g., where the function is increasing or decreasing, linear or nonlinear). Sketch or determine a graph that exhibits the qualitative features of a function that has been described verbally. |
| Understand the concept of a function and use function notation. | CC.2.2.HS.C.1  Use the concept and notation of functions to interpret and apply them in terms of their context. | A1.2.1.1.1, A1.2.1.1.2, A1.2.1.1.3, A1.2.2.1.1, A1.2.2.1.2, A1.2.2.1.3, A1.2.2.1.4  F.IF. 1. Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then f(x) denotes the output of f corresponding to the input x. The graph of f is the graph of the equation y = f(x).  2. Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.  3. Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. For example, the Fibonacci sequence is defined recursively by f(0) = f(1) = 1, f(n+1) = f(n) + f(n-1) for n ≥ 1. Learn as general principle; focus on linear and exponential and on arithmetic and geometric sequences. |
| Linear Functions | Define, evaluate, and compare functions.  Define, interpret, and compare functions displayed algebraically, graphically, numerically in tables, or by verbal descriptions. | CC.2.2.8.C.1 Define, evaluate, and compare functions. | M08.B-F.1.1.2 Compare properties of two functions, each represented in a different way (i.e., algebraically, graphically, numerically in tables, or by verbal descriptions). Example: Given a linear function represented by a table of values and a linear function represented by an algebraic expression, determine which function has the greater rate of change.  M08.B-F.1.1.3 Interpret the equation y = mx + b as defining a linear function whose graph is a straight line; give examples of functions that are not linear. |
| Use functions to model relationships between quantities.  Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values. | CC.2.2.8.C.2  Use concepts of functions to model relationships between quantities. | M08.B-F.2.1.1 Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two (x, y) values, including reading these from a table or from a graph. Interpret the rate of change and initial value of a linear function in terms of the situation it models and in terms of its graph or a table of values. |
| Interpret the structure of expressions. | CC.2.2.HS.D.1  Interpret the structure of expressions to represent a quantity in terms of its context. | A1.1.1.5.1, A1.1.1.5.2, A1.1.1.5.3  A.SSE.  1. Interpret expressions that represent a quantity in terms of its context. a. Interpret parts of an expression, such as terms, factors, and coefficients. b. Interpret complicated expressions by viewing one or more of their parts as a single entity. For example, interpret P(1+r)n as the product of P and a factor not depending on P.  2. Use the structure of an expression to identify ways to rewrite it. For example, see x 4 – y 4 as (x 2 ) 2 – (y 2 ) 2, thus recognizing it as a difference of squares that can be factored as (x2 – y 2)(x 2 + y 2). |
| Create equations that describe numbers or relationships. | CC.2.2.HS.D.7  Create and graph equations or inequalities to describe numbers or relationships. | A1.1.2.1.1, A1.1.2.1.2, A1.1.2.1.3, A1.1.2.2.1, A1.1.2.2.2, A1.1.3.1.1, A1.1.3.1.2, A1.1.3.1.3, A1.1.3.2.1, A1.1.3.2.2  A.CED. 1. Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear and quadratic functions, and simple rational and exponential functions. 2. Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales. 3. Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context. For example, represent inequalities describing nutritional and cost constraints on combinations of different foods. 4. Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. For example, rearrange Ohm’s law V = IR to highlight resistance R. Linear, quadratic, and exponential (integer inputs only); for A.CED.3 linear only. |
| Represent and solve equations and inequalities graphically. | CC.2.2.HS.D.10  Represent, solve, and interpret equations/inequalities and systems of equations/inequalities algebraically and graphically. | A1.1.2.1.1, A1.1.2.1.2, A1.1.2.1.3, A1.1.2.2.1, A1.1.2.2.2, A1.1.3.1.1, A1.1.3.1.2, A1.1.3.1.3, A1.1.3.2.1, A1.1.3.2.2  A.REI.  3. Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.  4. Solve quadratic equations in one variable. a. Use the method of completing the square to transform any quadratic equation in x into an equation of the form (x – p) 2 = q that has the same solutions. Derive the quadratic formula from this form. b. Solve quadratic equations by inspection (e.g., for x 2 = 49), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as a ± bi for real numbers a and b.  5. Prove that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions.  6. Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.  7. Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically. For example, find the points of intersection between the line y = –3x and the circle x 2 + y 2 = 3.  10. Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).  11. Explain why the x-coordinates of the points where the graphs of the equations y = f(x) and y = g(x) intersect are the solutions of the equation f(x) = g(x); find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where f(x) and/or g(x) are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.  12. Graph the solutions to a linear inequality in two variables as a half-plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes. |
| Build a function that models a relationship between two quantities. | CC.2.2.HS.C.3  Write functions or sequences that model relationships between two quantities. | A1.1.2.1.1, A1.1.2.1.2, A1.1.2.1.3, A1.2.1.1.1, A1.2.1.1.2, A1.2.1.1.3, A1.2.1.2.1, A1.2.1.2.2, A1.2.2.1.3, A1.2.2.1.4  F.BF. 1. Write a function that describes a relationship between two quantities. a. Determine an explicit expression, a recursive process, or steps for calculation from a context. b. Combine standard function types using arithmetic operations. For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model.  2. Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms. |
| Build new functions from existing functions. | CC.2.2.HS.C.4  Interpret the effects transformations have on functions and find the inverses of functions. | A1.2.1.2.1, A1.2.1.2.2  F.BF. 3. Identify the effect on the graph of replacing f(x) by f(x) + k, k f(x), f(kx), and f(x + k) for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them. 4. Find inverse functions. a. Solve an equation of the form f(x) = c for a simple function f that has an inverse and write an expression for the inverse. For example, f(x) =2 x3 or f(x) = (x+1)/(x–1) for x ≠ 1. |
| Interpret functions that arise in applications in terms of the context. | CC.2.2.HS.C.1  Use the concept and notation of functions to interpret and apply them in terms of their context. | A1.2.1.1.1, A1.2.1.1.2, A1.2.1.1.3, A1.2.1.2.1, A1.2.1.2.2, A1.2.2.1.1  F.IF. 1. Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then f(x) denotes the output of f corresponding to the input x. The graph of f is the graph of the equation y = f(x). 2. Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context. 3. Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. For example, the Fibonacci sequence is defined recursively by f(0) = f(1) = 1, f(n+1) = f(n) + f(n-1) for n ≥ 1. Learn as general principle; focus on linear and exponential and on arithmetic and geometric sequences. |
| Analyze functions using different representations. | CC.2.2.HS.C.2  Graph and analyze functions and use their properties to make connections between the different representations | A1.2.1.1.1, A1.2.1.1.2, A1.2.1.1.3, A1.2.1.2.1, A1.2.1.2.2, A1.2.2.1.1  F.IF. 4. For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity. 5. Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. For example, if the function h(n) gives the number of personhours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function. 6. Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph. 7. Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. a. Graph linear and quadratic functions and show intercepts, maxima, and minima. b. Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions. e. Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude. 8. Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function. a. Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context. b. Use the properties of exponents to interpret expressions for exponential functions. For example, identify percent rate of change in functions such as y = (1.02)t, y = (0.97)t, y = (1.01)12t, y = (1.2)t/10, and classify them as representing exponential growth or decay |
| Construct and compare linear and exponential models and solve problems. | CC.2.2.HS.C.5  Construct and compare linear, quadratic, and exponential models to solve problems. | A1.2.2.1.1, A1.2.2.1.2, A1.2.2.1.3, A1.2.2.1.4  F.LE.  1. Distinguish between situations that can be modeled with linear functions and with exponential functions. a. Prove that linear functions grow by equal differences over equal intervals, and that exponential functions grow by equal factors over equal intervals. b. Recognize situations in which one quantity changes at a constant rate per unit interval relative to another. c. Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another  2. Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).  3. Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratic ally, or (more generally) as a polynomial function. |
| Interpret expressions for functions in terms of the situation they model. | CC.2.2.HS.C.6  Interpret functions in terms of the situations they model. | A1.2.1.2.1, A1.2.2.1.2, A1.2.2.1.3, A1.2.2.2.1  F.LE.  5. Interpret the parameters in a linear or exponential function in terms of a context. |
| Linear Relationship in Geometric Figures | Define, evaluate, and compare functions.  Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values. | CC.2.2.8.C.1Define, evaluate, and compare functions. | M08.B-F.1.1.3 Interpret the equation y = mx + b as defining a linear function whose graph is a straight line; give examples of functions that are not linear. |
| Use functions to model relationships between quantities. | CC.2.2.8.C.2  Use concepts of functions to model relationships between quantities. | M08.B-F.2.1.1 Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two (x, y) values, including reading these from a table or from a graph. Interpret the rate of change and initial value of a linear function in terms of the situation it models and in terms of its graph or a table of values. |
| Experiment with transformations in the plane. | CC.2.3.HS.A.1  Use geometric figures and their properties to represent transformations in the plane. | G.1.3.1.1, G.1.3.1.2 |
| Use coordinates to prove simple geometric theorems algebraically. | CC.2.3.HS.A.11  Apply coordinate geometry to prove simple geometric theorems algebraically. | G.2.1.2.1, G.2.1.2.2, G.2.1.2.3 |
| Exponential Functions | Define, evaluate, and compare functions. | CC.2.2.8.C.1  Define, evaluate, and compare functions. | M08.B-F.1.1.2 Compare properties of two functions, each represented in a different way (i.e., algebraically, graphically, numerically in tables, or by verbal descriptions). Example: Given a linear function represented by a table of values and a linear function represented by an algebraic expression, determine which function has the greater rate of change. |
| Interpret the structure of expressions. | CC.2.2.HS.D.1  Interpret the structure of expressions to represent a quantity in terms of its context.  A-SSE.1a, b | A1.1.1.5.1, A1.1.1.5.2, A1.1.1.5.3  A.SSE.  1. Interpret expressions that represent a quantity in terms of its context. a. Interpret parts of an expression, such as terms, factors, and coefficients. b. Interpret complicated expressions by viewing one or more of their parts as a single entity. For example, interpret P(1+r)n as the product of P and a factor not depending on P.  2. Use the structure of an expression to identify ways to rewrite it. For example, see x 4 – y 4 as (x 2 ) 2 – (y 2 ) 2, thus recognizing it as a difference of squares that can be factored as (x2 – y 2)(x 2 + y 2). |
| Create equations that describe numbers or relationships. | CC.2.2.HS.D.7  Create and graph equations or inequalities to describe numbers or relationships. | A1.1.2.1.1, A1.1.2.1.2, A1.1.2.1.3, A1.1.2.2.1, A1.1.2.2.2, A1.1.3.1.1, A1.1.3.1.2, A1.1.3.1.3, A1.1.3.2.1, A1.1.3.2.2  A.CED.  1. Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear and quadratic functions, and simple rational and exponential functions. 2. Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.  3. Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context. For example, represent inequalities describing nutritional and cost constraints on combinations of different foods.  4. Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. For example, rearrange Ohm’s law V = IR to highlight resistance R. Linear, quadratic, and exponential (integer inputs only); for A.CED.3 linear only. |
| Represent and solve equations and inequalities graphically. | CC.2.2.HS.D.10  Represent, solve, and interpret equations/inequalities and systems of equations/inequalities algebraically and graphically. | A1.1.2.1.1, A1.1.2.1.2, A1.1.2.1.3, A1.1.2.2.1, A1.1.2.2.2, A1.1.3.1.1, A1.1.3.1.2, A1.1.3.1.3, A1.1.3.2.1, A1.1.3.2.2  A.REI.  3. Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.  4. Solve quadratic equations in one variable. a. Use the method of completing the square to transform any quadratic equation in x into an equation of the form (x – p) 2 = q that has the same solutions. Derive the quadratic formula from this form. b. Solve quadratic equations by inspection (e.g., for x 2 = 49), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as a ± bi for real numbers a and b.  5. Prove that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions.  6. Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.  7. Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically. For example, find the points of intersection between the line y = –3x and the circle x 2 + y 2 = 3.  10. Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).  11. Explain why the x-coordinates of the points where the graphs of the equations y = f(x) and y = g(x) intersect are the solutions of the equation f(x) = g(x); find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where f(x) and/or g(x) are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.  12. Graph the solutions to a linear inequality in two variables as a half-plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes. |
| Build a function that models a relationship between two quantities. | CC.2.2.HS.C.3  Write functions or sequences that model relationships between two quantities. | A1.1.2.1.1, A1.1.2.1.2, A1.1.2.1.3, A1.2.1.1.1, A1.2.1.1.2, A1.2.1.1.3, A1.2.1.2.1, A1.2.1.2.2, A1.2.2.1.3, A1.2.2.1.4  F.BF. 1. Write a function that describes a relationship between two quantities. a. Determine an explicit expression, a recursive process, or steps for calculation from a context. b. Combine standard function types using arithmetic operations. For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model. 2. Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms. |
| Build new functions from existing functions. | CC.2.2.HS.C.4  Interpret the effects transformations have on functions and find the inverses of functions. | A1.2.1.2.1, A1.2.1.2.2  F.BF. 3. Identify the effect on the graph of replacing f(x) by f(x) + k, k f(x), f(kx), and f(x + k) for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them. 4. Find inverse functions. a. Solve an equation of the form f(x) = c for a simple function f that has an inverse and write an expression for the inverse. For example, f(x) =2 x3 or f(x) = (x+1)/(x–1) for x ≠ 1. |
| Interpret functions that arise in applications in terms of the context. | CC.2.2.HS.C.1  Use the concept and notation of functions to interpret and apply them in terms of their context. | A1.2.1.1.1, A1.2.1.1.2, A1.2.1.1.3, A1.2.1.2.1, A1.2.1.2.2, A1.2.2.1.1  F.IF. 1. Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then f(x) denotes the output of f corresponding to the input x. The graph of f is the graph of the equation y = f(x). 2. Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context. 3. Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. For example, the Fibonacci sequence is defined recursively by f(0) = f(1) = 1, f(n+1) = f(n) + f(n-1) for n ≥ 1. Learn as general principle; focus on linear and exponential and on arithmetic and geometric sequences |
| Analyze functions using different representations. | CC.2.2.HS.C.2  Graph and analyze functions and use their properties to make connections between the different representations. | A1.2.1.1.1, A1.2.1.1.2, A1.2.1.1.3, A1.2.1.2.1, A1.2.1.2.2, A1.2.2.1.1 |
| Construct and compare linear and exponential models and solve problems. | CC.2.2.HS.C.5  Construct and compare linear, quadratic, and exponential models to solve problems. | A1.2.2.1.1, A1.2.2.1.2, A1.2.2.1.3, A1.2.2.1.4 |
| Interpret expressions for functions in terms of the situation they model. | CC.2.2.HS.C.6  Interpret functions in terms of the situations they model. | A1.2.1.2.1, A1.2.2.1.2, A1.2.2.1.3, A1.2.2.2.1, |
| Working with Polynomials | Perform arithmetic operations on polynomials. | CC.2.2.HS.D.3  Extend the knowledge of arithmetic operations and apply to polynomials.  A-APR.1 | A1.1.1.5.1, A1.1.1.5.2, A1.1.1.5.3 |
| Interpret the structure of expressions. | CC.2.2.HS.D.1  Interpret the structure of expressions to represent a quantity in terms of its context. | A1.1.1.A1.1.1.5.2, A1.1.1.5.3 |
| Quadratic Functions | Define, evaluate, and compare functions. | CC.2.2.8.C.1 Define, evaluate, and compare functions. | M08.B-F.1.1.1 Determine whether a relation is a function.  M08.B-F.1.1.2 Compare properties of two functions, each represented in a different way (i.e., algebraically, graphically, numerically in tables, or by verbal descriptions). Example: Given a linear function represented by a table of values and a linear function represented by an algebraic expression, determine which function has the greater rate of change.  M08.B-F.1.1.3 Interpret the equation y = mx + b as defining a linear function whose graph is a straight line; give examples of functions that are not linear. |
| Interpret the structure of expressions. | CC.2.2.HS.D.1  Interpret the structure of expressions to represent a quantity in terms of its context. | A1.1.1.5.1, A1.1.1.5.2, A1.1.1.5.3 |
| Write expressions in equivalent forms to solve problems. | CC.2.2.HS.D.2  Write expressions in equivalent forms to solve problems. | A1.1.1.5.1, A1.1.1.5.2, A1.1.1.5.3 |
| Create equations that describe numbers or relationships. | CC.2.2.HS.D.7  Create and graph equations or inequalities to describe numbers or relationships. | A1.1.2.1.1, A1.1.2.1.2, A1.1.2.1.3, A1.1.2.2.1, A1.1.2.2.2, A1.1.3.1.1, A1.1.3.1.2, A1.1.3.1.3, A1.1.3.2.1, A1.1.3.2.2 |
| Construct and compare linear and exponential models and solve problems. | CC.2.2.HS.C.5  Construct and compare linear, quadratic, and exponential models to solve problems. | A1.2.2.1.1, A1.2.2.1.2, A1.2.2.1.3, A1.2.2.1.4 |
| Interpret functions that arise in applications in terms of the context. | CC.2.2.HS.C.1  Use the concept and notation of functions to interpret and apply them in terms of their context. | A1.2.1.1.1, A1.2.1.1.2, A1.2.1.1.3, A1.2.2.1.1, A1.2.2.1.2, A1.2.2.1.3, A1.2.2.1.4 |
| Analyze functions using different representations. | CC.2.2.HS.C.2  Graph and analyze functions and use their properties to make connections between the different representations. | A1.2.1.1.1, A1.2.1.1.2, A1.2.1.1.3, A1.2.1.2.1, A1.2.1.2.2, A1.2.2.1.1 |
| Descriptive Statistics | Summarize, represent, and interpret data on a single count or measurement variable. | CC.2.4.HS.B.1  Summarize, represent, and interpret data on a single count or measurement variable. | A1.2.2.1.2, A1.2.3.1.1, A1.2.3.2.1, A1.2.3.2.2, A1.2.3.2.3  S.ID.  1. Represent data with plots on the real number line (dot plots, histograms, and box plots).  2. Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets.  3. Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers). |
| Bivariate Data | Investigate patterns of association in bivariate data. | CC.2.4.HS.B.1 Summarize, represent, and interpret data on a single count or measurement variable.  CC.2.4.HS.B.2  Summarize, represent, and interpret data on two categorical and quantitative variables. | A1.2.2.1.2, A1.2.3.1.1, A1.2.3.2.1, A1.2.3.2.2, A1.2.3.2.3, A1.2.1.1.1, A1.2.1.1.2, A1.2.1.1.3, A1.2.1.2.1, A1.2.1.2.2, A1.2.2.2.1  S.ID.  1. Represent data with plots on the real number line (dot plots, histograms, and box plots).  2. Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets.  3. Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers).  5. Summarize categorical data for two categories in two-way frequency tables. Interpret relative frequencies in the context of the data (including joint, marginal, and conditional relative frequencies). Recognize possible associations and trends in the data.  6. Represent data on two quantitative variables on a scatter plot, and describe how the variables are related. a. Fit a function to the data; use functions fitted to data to solve problems in the context of the data. Use given functions or choose a function suggested by the context. Emphasize linear, quadratic, and exponential models. b. Informally assess the fit of a function by plotting and analyzing residuals. c. Fit a linear function for a scatter plot that suggests a linear association. |
| Summarize, represent, and interpret data on two categorical and quantitative variables. | CC.2.4.HS.B.2  Summarize, represent, and interpret data on two categorical and quantitative variables. | A1.2.1.1.1, A1.2.1.1.2, A1.2.1.1.3, A1.2.1.2.1, A1.2.1.2.2, A1.2.2.2.1  S.ID. 5. Summarize categorical data for two categories in two-way frequency tables. Interpret relative frequencies in the context of the data (including joint, marginal, and conditional relative frequencies). Recognize possible associations and trends in the data. 6. Represent data on two quantitative variables on a scatter plot, and describe how the variables are related. a. Fit a function to the data; use functions fitted to data to solve problems in the context of the data. Use given functions or choose a function suggested by the context. Emphasize linear, quadratic, and exponential models. b. Informally assess the fit of a function by plotting and analyzing residuals. c. Fit a linear function for a scatter plot that suggests a linear association. |
| Interpret linear models. | CC.2.4.HS.B.3  Analyze linear models to make interpretations based on the data. | A1.2.3.3.1  S.ID.  7. Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data.  8. Compute (using technology) and interpret the correlation coefficient of a linear fit.  9. Distinguish between correlation and causation. |